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LETTER TO THE EDITOR

On the ballistic conductance of small contacts and its resonant structure: trumpet effect washes out resonant structure

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Abstract. Recent experimental results in 2D electron gas GaAs/GaAlAs structures seem to show that the ballistic electron conductance of small contacts, to a good approximation, is 'quantised'. In this Letter we present calculations for general geometries of the contacts which describe the experimental data reasonably well. Our calculations show resonant scattering superimposed on the quantised conductance for some particular geometries. In general the resonances tend to be washed out and most probably in experimental geometries the resonances should not be present.

In recent work on high-mobility 2DEG in GaAs–GaAlAs heterostructures [1, 2], the conductance (G) of ballistic point contacts has been shown to exhibit a 'quantised' behaviour as a function of the width of the contact, W (see figure 1). The resistance/conductance of these contacts was studied theoretically in [3], which showed that in the classical limit, when the electrons going through the contact are treated like bullets and both the length of the contact, l, and the width, W, are smaller than the inelastic electron mean free path, the value of G is

$$G = 2G_0 W \tag{1}$$

where $G_0 = 1/R_0$ with $R_0 = 2h/e^2 \sim 12{,}900\ \Omega$ being the quantum of resistance, and W is defined in units of the Fermi wavelength, λ. However, the calculations in [3] did not introduce quantum interferences that should be present when the electrons diffract with small contacts, i.e. W of a few wavelengths. Quantum calculations of the elastic resistance of small scanning tunnelling microscopy (STM) point contacts discussed in a talk on oscillatory behaviour in the quantum elastic resistance of small contacts [4] clearly show that the resistances presented quantum oscillations or plateaus as a function of the contact width. These calculations were done for contacts of short length $l < \lambda/2$ and the shapes of the oscillations in the conductance depended also on the given geometrical shape of the contact [4, 5].

In this work we present quantum exact calculations for the ballistic conductance of small contacts of general geometry. In order to be more explicit, the inset of figure 1 shows the geometry of the contact and the reservoirs of electrons of Fermi wavelengths λ_1, λ_2 and λ_t describing the reservoirs and the contact respectively (in our case $\lambda_1 = \lambda_2 = \lambda_t$, i.e. the 2DEG has the same Fermi energy in all 2D space). The reservoirs have width

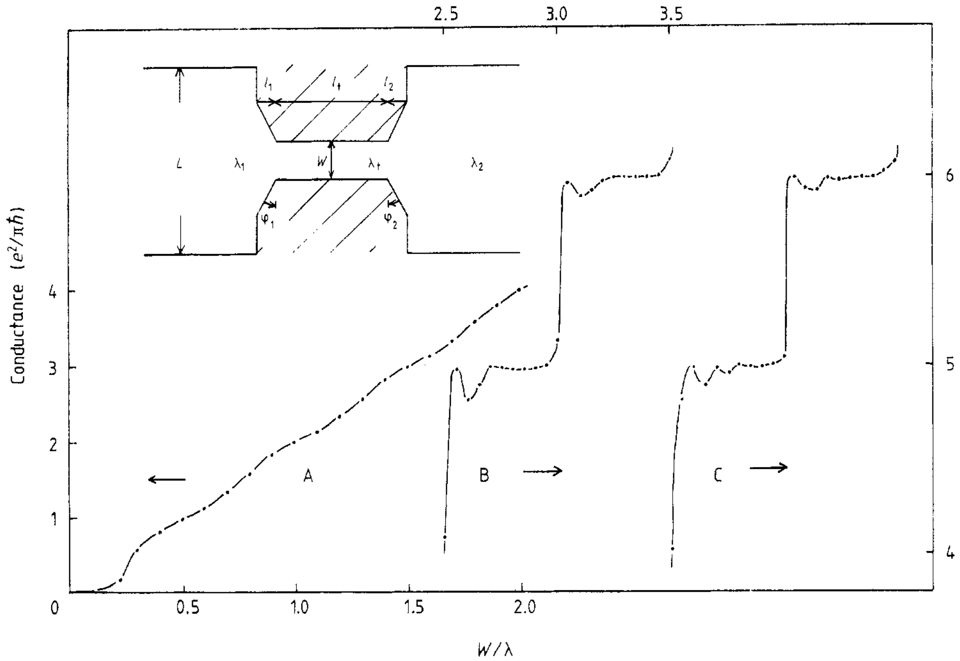

Figure 1. The inset describes the geometry of the contact and the reservoirs. Notice in our geometry the two *trumpets* connecting the reservoirs and the contacts of width W. These trumpets are defined by l_1, l_2, φ_1 and φ_2. Curve A: oscillatory character of the conductance versus W for $l_t = 0.001$ and no trumpets; curves B and C: the same as in curve A but for l_t = 2 and 5 respectively. In these cases well defined steps and plateaus develop as well as a superimposed resonant structure. In all cases $L = 10$ and no appreciable changes were observed for larger values of L. The left-hand and lower axis labels apply to curve A, and the rigid-hand and upper axis labels to curves B and C.

$L \gg W$ and the length of the contact is $l = l_1 + l_t + l_2$, where l_1 and l_2 describe the length of the *trumpets* ending the contact with angles φ_1 and φ_2. Also, l_t is the length where the contact has constant width W. The contacts are defined by applying a negative voltage to the gate described by the shaded region that creates an infinite large repulsive potential to the electrons in the 2DEG.

We have performed quantum mechanical calculations to obtain the values of G as a function of W for different values of the parameters of the inset in figure 1. The results are obtained by solving Schrödinger's equation with the boundaries defined by the contact constriction and the reservoir walls by a similar method to the one used in [6] to calculate quantum elastic reinstances of interfaces. In this case because the reservoirs have dimension L we have expanded the solution in the reservoirs into series of sines and cosines vanishing at the walls of the reservoirs. The solution in the constriction (l_t, W) is also expanded in sine and cosine owing to the characteristic modes of the width W. At the trumpets, i.e. the opening regions of the contacts, defined by l_1, l_2, φ_1 and φ_2, the solution is expanded into sine and cosine linear combinations of the reservoir 1 and 2 respectively. This solution in the trumpets is within the Rayleigh assumption [7] and the good results should be checked against the unitary rules in the reflected and transmitted intensity through the contact. The solution in all space is then obtained by matching the wavefunction reservoir 1, the constriction and reservoir 2, and at the same time this wavefunction has to vanish at the boundaries defined by the contact. The conductance is obtained by

$$G = G_0 \sum_i T_i \tag{2}$$

where T_i is the total transmitivity for the electron i impinging the contact and going through it. Also

$$T_i = \sum_i T_{ij} \tag{3}$$

where T_{ij} is the transmittivity carried out by the j-component of the propagating wave of the transmitted electron (see [6]). The expansion of the wavefunction has a finite number of propagating terms N_p and an infinite number of evanescent waves that has to be truncated to N_e terms, and the convergence has to be studied as a function of these evanescent terms that play a very important role in the convergent solutions. The index i in equation (2) runs over all states at the Fermi level.

In what follows all our parameters are given in units of $\lambda = 1$. Figure 1 shows results for three characteristic cases. In all of them we have assumed that the contact is defined by a perfect tube of constant width ($l_1 = l_2 = 0$). Figure 1, curve A, shows the case of $l_t = 0.001$, for which only very weak and small oscillations are observed of period 0.5 and the average slope of the curve is 2, in agreement with formula (1); i.e. for a pure flat contact the quantisation is difficult to define. In figure 1, curve B, we present calculations for $l_t = 2$ and we can now observe plateaus in the conductance at integral values of G with periods of 0.5; however, we also notice the apparition of a resonant structure superimposed on the plateaus. This resonant structure appears only when the value of l_t is larger than a critical value of about 0.7 in agreement with our previous results [4, 5] where we did not find resonances when $l_t < 0.5$. These resonances are well understood in terms of the resonant character of the transmittivity of a step of length l_t and height V [8]; here, the height of the step is the energy of the different propagating modes n in the constriction of energy $(\hbar^2/2m^*)(n\pi/W)^2$, m^* being the effective mass of the electron. Figure 1, curve C, shows similar calculations to those of figure 1, curve B, but with $l_t = 5$, and certainly we observe that the number of resonances increases. The jumps or 'quantised' steps in the conductance can also be understood in terms of the number of propagating or conducting modes owing to the contact that are defined by int($2W$). The conductance of wires and tubes has been clearly discussed in [9].

However, the above-calculated cases show resonances that are quite fictitious, because for real-life devices the width of the contact should change smoothly in the reservoir regions because the depletion of electrons by applying the gate voltage should vary slowly. In other words the real contacts should show trumpets that we simulate by the angles φ_1 and φ_2 in the inset of figure 1.

In figure 2 we analyse the behaviour of the resonances in figure 1, curve B for $l_t = 2$ when we add trumpets of different length, and in order to keep the parameters under control we take in all cases $\varphi_1 = \varphi_2 = 30°$. Figure 2, curve A, represents an augmentation of figure 1, curve B, to be compared with the following cases. Figure 2, curve B, shows the resonant structure for a trumpet connecting reservoir 1 and length $l_1 = 0.5$; figure 2, curve C, presents the resonant structure with two trumpets of length $l_1 = l_2 = 0.5$. It can be observed that the resonant structure remains practically unchanged for trumpet lengths smaller than 0.5. However, this is not the case when the trumpets are longer. For $l_1 = l_2 = 0.75$ and $l_1 = l_2 = 1$ corresponding to figure 2, curves D and E, the strong

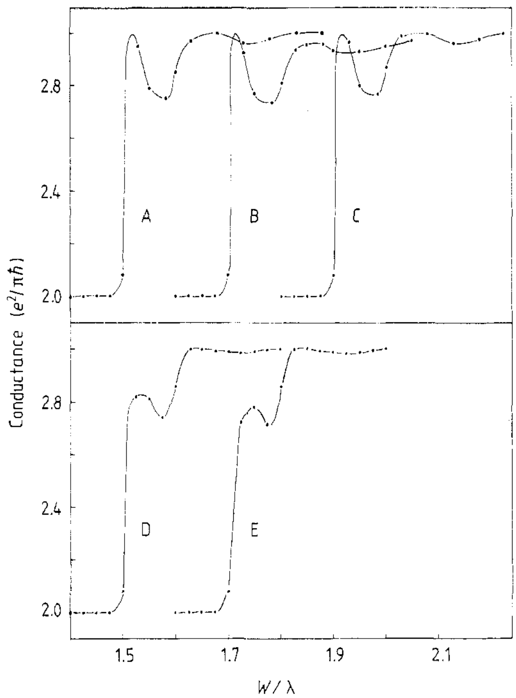

Figure 2. Curve A: conductance versus W for the case of figure 1, curve B; curves B, C, D and E: the same calculations as for curve A but with introduction of trumpets in the geometry of inset 1, $\varphi_1 = \varphi_2 = 30°$. For curve B, $l_1 = 0.5$ and $l_2 = 0$; curve C, $l_1 = l_2 = 0.5$; curve D, $l_1 = l_2 = 0.75$; curve E, $l_1 = l_2 = 1$. Notice the *trumpet effect* is to wash out the resonant structure; compare curve A with curves D and E.

resonance together with the weak one are clearly washed out and tend to disappear in such a way that the trend is to smooth out the jump in the conductance step. We have tried to increase the length of the trumpets for l_1 and l_2 larger than 1, but our results do not converge. Physically it is very easy to understand this disappearing behaviour of the resonance for long trumpets (*trumpet effect*) and it is simply related to the behaviour of the transmitivity of a particle through a smooth step. Now the step in the potential V mentioned above takes place in a wavelength distance and the resonant structure in the transmitivity of the particle disappears [8].

In conclusion, we have calculated the elastic ballistic conductance/resistance of electrons through small contacts. Our results show that for short contacts the conductance shows a periodic oscillatory behaviour, that transforms into plateaus and steps of height $e^2/2h$ when the length of the contact is of the order of λ. The plateaus also show a superimposed resonant structure for the cases in which the interfaces between the contact and the reservoirs have sharp edges. However, these resonances disappear for general geometries by introducing what we have called the trumpet effect. In our opinion, experimental results should not show resonances, because the electron depletion between the constriction defining the contact and the reservoir should vary slowly and in a Fermi wavelength.

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